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Time-fractional Thermoelastic Deflection of a Thin Clamped Circular Plate

Hamna Mirza
Department of Mathematics,
Swamy Vivekanand Govt. P.G. College, Harda (Madhya Pradesh), India.

(Corresponding author: Hamna Mirza)
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ABSTRACT: In this paper, consider a thin clamped circular plate occupying the space D: $0 \le r \le a$, $0 \le z \le h$, with the stated boundary conditions. The temperature distribution and thermal deflection have been determined on upper plane surface of circular plate in the context of fractional-order theory of thermoelasticity by quasi-static approach. The finite Hankel transform and Laplace transform techniques are used to find the solution. The results obtained are in terms of Bessel' function of in the form of infinite series. Numerical computation has been done for temperature and deflection and illustrated graphically.

Keywords*:* Fractional Order, circular plate, Integral transform, Mittag-Leffler function, thermal stresses.

I. INTRODUCTION

Nowacki [2] has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge, respectively. Roychaudhari [3] has succeeded in determining the transient temperature along the circumference of circular upper face with lower face is at zero temperature and the fixed circular edge thermally insulated. Wankhede [4] has determined the quasi-static thermal stresses in circular plate, subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. Ishihara *et al.*, [1] has considered a circular plate and discussed the transient thermoelastic-plastic bending problem, making use of the strain increment theorem. Ezzat and El-Karamany [9] derived a new theory of thermoelasticity by using the methodology of fractional calculus. Lamba and Khobragade (2012) [13] studied the three-dimensional inverse transient thermoelastic problem for a thin rectangular object within the context of the theory of generalized thermoelasticity. Lamba and Khobragade (2012) [14] studied the uncoupled thermoelastic response of thick cylinder of length 2h in which heat sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. Hendy *et al.*, (2017) [10] constructed a new mathematical model of magneto-thermoelasticity theory in the context of a new consideration of heat conduction law with time-fractional order. Raslan (2014) [11] applied the model of fractional magneto-thermoelasticity to a one-dimensional thermal shock problem for a functionally graded half-space subjected to traction free surface due to arbitrary thermal loading. Raslan [11] applied the fractional order theory of thermoelasticity to a 1D problem of an infinitely long cylindrical cavity. Sherief and Abd El-Latief [12] applied the fractional order theory of thermoelasticity to a 1D thermal shock problem for a half-space and simplified by using Laplace transform technique.

Recently, Khobragade and Lamba (2019) [15] determined thermal deflection and stresses of a circular disk subjected to axisymmetric and partially distributed heat supply on upper surface while the lower surface is kept thermally insulated by application of fractional order theory. Khobragade and Lamba (2019) [16] analyzed the temperature distribution, displacement, thermal stresses function and deflection on outer curved surface of a solid circular cylinder by application of fractional order theory.

In this chapter, an attempt is made to determine the unknown temperature, temperature distribution, and thermal deflection on upper plane surface of a thin clamped circular plate occupying the space D: $0 \le r \le a$, $0 \le z \le h$ with the stated boundary conditions. The finite Hankel transform and Laplace transform techniques have been used to obtain the solution of the problem.

II. PROBLEM FORMULATION

Consider an isotropic circular plate of thickness h occupying the space
$D: 0 \le r \le a,\ 0 \le z \le h$. The differential equation governing the displacement function $U(r, z, t)$ as [2] is

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}=(1+\nu)a_t T$$

(1) with $U_r = 0$ at $r = a$ (2) where v and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and $T(r, z, t)$ is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial^\alpha T}{\partial t^\alpha} \tag{3}$$

subject to the initial condition

$$T(r,z,0)=0 \tag{4}$$

the boundary conditions

$$T(a,z,t)=0 \tag{5}$$

$$\left[T(r,z,t)\right]_{z=h}=g(r)\delta(t) \tag{6}$$

$$\left[T(r,z,t)+c\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0}=u(r)\delta(t) \tag{7}$$

where k is the thermal diffusivity of the material of the plate and c is arbitrary constant. The Eqns. (1) to (7) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

To obtain the expression for the temperature function $T(r, z, t)$, we define the finite Hankel transform as

If $f(x)$ satisfies Dirichlet's conditions in the interval $(0, a)$ then its finite Hankel transform in that range is defined to be

$$\bar{f}_\mu(\xi_i)=\int_0^a x\,f(x)\,J_\mu(x\xi_i)\,dx \tag{8}$$

where ξ_i is the root of the transcendental equation

$$J_\mu(a\,\xi_i)=0 \tag{9}$$

then at any point of $(0, a)$ at which the function $f(x)$ is continuous,

$$f(x)=\frac{2}{a^2}\sum_i \bar{f}_\mu(\xi_i)\frac{J_\mu(x\xi_i)}{\left[J'_\mu(a\xi_i)\right]^2} \tag{10}$$

where the sum is taken over all the positive roots of the Eqn. (9)

Apply integral transform defined in Eqn. (9) to the Eqns. (3) to (7) and using (5), one obtains

$$\frac{d^2\bar{T}}{dz^2}-\lambda_n{}^2\,\bar{T}=\frac{1}{k}\frac{d^\alpha \bar{T}}{dt^\alpha} \tag{11}$$

$$\bar{T}(\lambda_n, z,0)=0 \tag{12}$$

$$\left[\bar{T}(\lambda_n, z,t)\right]_{z=h}=\bar{g}(\lambda_n)\delta(t) \tag{13}$$

$$\left[\bar{T}(\lambda_n,z,t)+c\frac{d\bar{T}(\lambda_n,z,t)}{dz}\right]_{z=0}=\bar{u}(\lambda)\,\delta(t) \tag{14}$$

where $\bar{T}$ denotes the finite Hankel transform of T and λ_n is the Hankel transform parameter.

In Eq. (12), $(\partial^\alpha T/\partial t^\alpha)$ is the Caputo fractional derivative [5]-[7] as

$$\frac{d^\alpha f(t)}{dt^\alpha}=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n}d\tau, & n-1<\alpha<n,\\[2ex] \dfrac{d^n f(\tau)}{dt^n}, & \alpha=n\end{cases} \tag{15}$$

with the following Laplace transform rule

$$L\left\{\frac{d^{\alpha} f(t)}{dt^{\alpha}}\right\} = s^{\alpha} L\{\bar{f}(s)\} - \sum_{k=0}^{n-1} f^{(k)}(0^{+})\, s^{\alpha-1-k}, \quad n-1 < \alpha < n. \tag{16}$$

in which s is the transform parameter.

Applying Laplace transform stated in (16) to the equations (11) and using (12) to (14), one obtains

$$\frac{d^2 \overline{T}^{*}}{dz^2} - q^2 \overline{T}^{*} = 0 \tag{17}$$

where $q^2 = \lambda_n{}^2 + \frac{1}{k}\left[s^{\alpha} L\{\theta\} - \sum_{r=0}^{r=n-1} \theta^{(r)}(0^{+}) s^{\alpha-1-r} \right]$

$$\left[\overline{T}^{*}(\lambda_n, z, s) \right]_{z=h} = \overline{g}^{*}(\lambda_n) \tag{18}$$

$$\left[\overline{T}^{*}(\lambda_n, z, s) + c\,\frac{d\overline{T}^{*}(\lambda_n, z, s)}{dz} \right]_{z=0} = \overline{u}^{*}(\lambda_n) \tag{19}$$

where $\overline{T}^{*}$ denotes the Laplace transform of $\overline{T}$ and s is the Laplace transform parameter.
The equation (17) is a second order differential equation whose solution is given by

$$\overline{T}^{*}(\lambda_n, z, s) = A e^{qz} + B e^{-qz} \tag{20}$$

where A and B are arbitrary constants.
Using (3.8) and (3.9) in (3.10), we obtain

$$A = \frac{-1}{(1+cq)(e^{q\xi} - e^{-q\xi})}\left[\overline{u}^{*}(\lambda_n) e^{-q\xi} - \overline{f}^{*}(\lambda_n) \right]$$

And $B = \frac{1}{(1-cq)(e^{q\xi} - e^{-q\xi})}\left[\overline{u}^{*}(\lambda_n) e^{q\xi} - \overline{f}^{*}(\lambda_n) \right]$

Substituting the values of A and B in Eqn. (20) and then applying inversion of Laplace transform and finite Hankel transform, one obtains

$$\begin{aligned} T(r,z,t) &= \frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2) J_1{}^2(\lambda_n a)} \\ &\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi z}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi z}{\xi}\right) \right] \\ &\times \int_0^t \overline{f}(\lambda_n, t')\, E_{\alpha}\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^{\alpha} - t'^{\alpha}) \right) - \frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2) J_1{}^2(\lambda_n a)} \\ &\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi(z-\xi)}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(z-\xi)}{\xi}\right) \right] \\ &\times \int_0^t \overline{u}(\lambda_n, t')\, E_{\alpha}\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^{\alpha} - t'^{\alpha}) \right) \end{aligned} \tag{21}$$

where $L^{-1}\left[\frac{1}{s^{\alpha} + k(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2})} \right] = E_{\alpha}\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2} \right)(t^{\alpha} - t'^{\alpha}) \right)$

Here $E_{\alpha}(.)$ represents the Mittag–Leffler function.

$$\begin{aligned} g(r,t) &= \frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2) J_1{}^2(\lambda_n a)} \\ &\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi h}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi h}{\xi}\right) \right] \end{aligned}$$

$$\times\int_0^t \bar{f}(\lambda_n,t')\,E_\alpha\left(-k\left(\lambda_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\frac{4k\pi(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}\times\sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi(h-\xi)}{\xi}\right)-\left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(h-\xi)}{\xi}\right)\right]$$

$$\times\int_0^t \bar{u}(\lambda_n,t')\,E_\alpha\left(-k\left(\lambda_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right) \tag{22}$$

where λ_n are the positive roots of the equation $J_0(\lambda_n a)=0$.

Quasi-Static Thermal Deflection: The differential equation satisfied by the deflection $w(r, t)$ is

$$D\nabla_1{}^4 w=\frac{-\nabla_1{}^2 M_T}{1-\nu} \tag{23}$$

where ν is the Poisson's ratio of the plate material, M_T denote the thermal momentum of the plate and D denote the flexural rigidity,

where $$\nabla_1{}^2=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}$$

Since the edge of the circular plate is fixed and clamped,

$$w=\left[\frac{\partial w}{\partial r}\right]_{r=a}=0 \tag{24}$$

We assume that the solution of Eqn. (23) satisfying Eqn. (24) as

$$w(r,t)=\sum_{n=1}^{\infty} c_n(t)\left[J_0(\lambda_n r)-J_0(\lambda_n a)\right] \tag{25}$$

The term M_T is defined as

$$M_T(r,t)=\alpha E\int_0^h zT(r,z,t)\,dz \tag{26}$$

Substituting the value of $T(r, z, t)$ from Eqn. (21) in Eqn. (26) one obtains

$$M_T(r,t)=\frac{4k\pi\alpha E(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{1}{(1-c^2\lambda_n{}^2)}\frac{J_0(\lambda_n r)}{J_1{}^2(\lambda_n a)}$$

$$\times\sum_{m=1}^{\infty}(-1)^{m+1}\left[1-(-1)^m\right]\left(\frac{h(z-\xi)}{\pi}\right)\times\int_0^t \bar{f}(\lambda_n,t')E_\alpha\left(-k\left(\lambda_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\frac{4k\pi\alpha E(1-c)}{a^2\xi^2}\sum_{n=1}^{\infty}\frac{1}{(1-c^2\lambda_n{}^2)}\frac{J_0(\lambda_n r)}{J_1{}^2(\lambda_n a)}\times\sum_{m=1}^{\infty}(-1)^{m+1}\left[1-(-1)^m\right]\frac{(\xi-z)}{\pi}$$

$$\times\int_0^t \bar{u}(\lambda_n,t')E_\alpha\left(-k\left(\lambda_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right) \tag{27}$$

Using the Eqns. (23), (27) and the result

$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]J_0(\lambda_n r)=-\lambda_n{}^2 J_0(\lambda_n r)$, one obtains

$$c_n(t)=\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2}\sum_{n=1}^{\infty}\frac{1}{\lambda_n{}^2(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}\times\sum_{m=1}^{\infty}(-1)^{m+1}\left[\frac{(z-\xi)h}{\pi}\right]\left[1-(-1)^m\right]$$

$$\times\int_0^t \bar{f}(\lambda_n,t')\,E_\alpha\left(-k\left(\lambda_n{}^2+\frac{m^2\pi^2}{\xi^2}\right)(t^\alpha-t'^\alpha)\right)$$

$$-\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2}\sum_{n=1}^{\infty}\frac{1}{\lambda_n{}^2(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}\times\sum_{m=1}^{\infty}(-1)^{m+1}\left[\frac{(\xi-z)h}{\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{u}(\lambda_n, t')\, E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right) \quad (28)$$

Substituting the value of $c_n(t)$ in equation (25), one obtains the quasi-static thermal deflection *w*(*r, t*) as

$$w(r,t) = \frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n{}^2(1-c^2\lambda_n{}^2)} \frac{[J_0(\lambda_n r) - J_0(\lambda_n a)]}{J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(z-\xi)h}{m\pi}\right]\left[1-(-1)^m\right] \times \int_0^t \bar{f}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$-\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n{}^2(1-c^2\lambda_n{}^2)} \frac{[J_0(\lambda_n r) - J_0(\lambda_n a)]}{J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(\xi-z)h}{m\pi}\right]\left[1-(-1)^m\right]$$

$$\times \int_0^t \bar{u}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right) \quad (29)$$

Special Case

Set $f(r)\,\delta(t) = (\xi + h)e^{\xi}(r-a)(1-e^{-t})$,

and $u(r)\,\delta(t) = (\xi + h)(r-a)(1-e^{-t})$ (30)

$$T(r,z,t) = \frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi z}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi z}{\xi}\right)\right]$$

$$\times \int_0^t \bar{f}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right) dt'$$

$$-\frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi(z-\xi)}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(z-\xi)}{\xi}\right)\right]$$

$$\times \int_0^t \bar{u}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

(31)

$$g(r,t) = \frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi h}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi h}{\xi}\right)\right] \times \int_0^t \bar{f}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right)$$

$$-\frac{4k\pi(1-c)}{a^2\xi^2} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{(1-c^2\lambda_n{}^2)J_1{}^2(\lambda_n a)} \times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\sin\left(\frac{m\pi(h-\xi)}{\xi}\right) - \left(\frac{m\pi}{\xi}\right)\cos\left(\frac{m\pi(h-\xi)}{\xi}\right)\right]$$

$$\times \int_0^t \bar{u}(\lambda_n, t') E_\alpha\left(-k\left(\lambda_n{}^2 + \frac{m^2\pi^2}{\xi^2}\right)(t^\alpha - t'^\alpha)\right) \quad (32)$$

$$w(r,t) = \frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n{}^2(1-c^2\lambda_n{}^2)} \frac{[J_0(\lambda_n r) - J_0(\lambda_n a)]}{J_1{}^2(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(z-\xi)h}{m\pi}\right]\left[1-(-1)^{m}\right]\times\int_{0}^{t}\bar{f}(\lambda_n,t')E_{\alpha}\left(-k\left(\lambda_n^{2}+\frac{m^2\pi^2}{\xi^2}\right)(t^{\alpha}-t'^{\alpha})\right)$$

$$-\frac{4k\pi\alpha E(1-c)}{D(1-\nu)a^2\xi^2}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^{2}(1-c^2\lambda_n^{2})}\frac{[J_0(\lambda_n r)-J_0(\lambda_n a)]}{J_1^{2}(\lambda_n a)}$$

$$\times \sum_{m=1}^{\infty} m(-1)^{m+1}\left[\frac{(\xi-z)h}{m\pi}\right]\left[1-(-1)^{m}\right]\times\int_{0}^{t}\bar{u}(\lambda_n,t')E_{\alpha}\left(-k\left(\lambda_n^{2}+\frac{m^2\pi^2}{\xi^2}\right)(t^{\alpha}-t'^{\alpha})\right) \quad (33)$$

IV. NUMERICAL RESULTS

The numerical calculation has been carried out for Copper (pure) circular plate with the material properties as thermal diffusivity $\kappa = 112.34\times10^{-6}$ m^2s^{-1} thermal conductivity $\lambda = 386$ (W/mK), density $\rho = 2954$ kg/m^3, Poisson's ratio $\upsilon = 0.35$, thermal expansion coefficient $\alpha = 16.5\times10^{-6}$ /°C and modulus of elasticity $E = 70$ GPa. The graphs are plotted for fractional-order parameter α = 0.5; 1; 1.5 and 2 depicting different conductivity and fixed time t =0.5. Figures *1,* depict the distributions of deflection along the radial direction for various values of fractional-order parameter α. The numerical calculation has been carried out in MATHEMATICA programing.

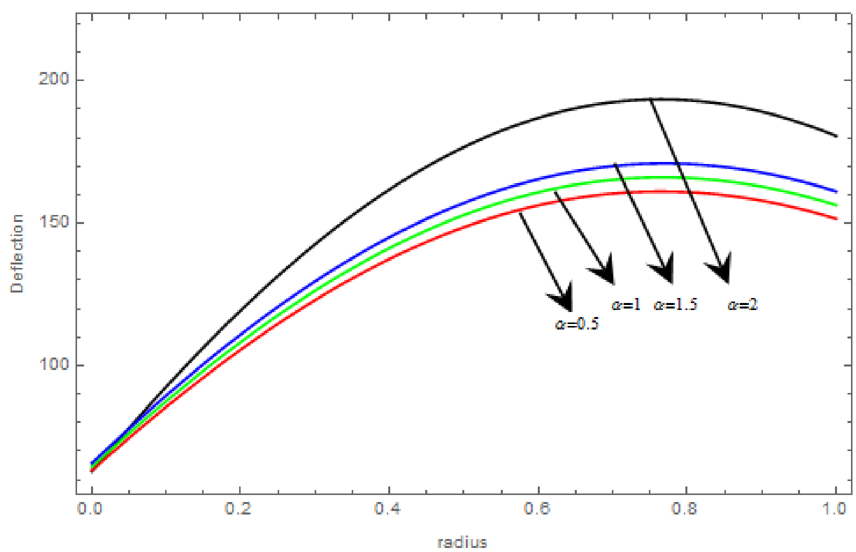

Fig. 1. Thermal Deflection along r- direction for different α.

Fig. 1, indicates the thermal deflection along the radial direction, it can be seen that there is a variation in the deflection in increasing order along the radius.

V. CONCLUSION

The temperature distribution and thermal deflection have been determined on upper plane surface of the circular plate with the aids of finite Hankel transform and Laplace transform techniques by fractional order theory of thermoelasticity. The results obtained are in terms of Bessel' function in the form of infinite series. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The expressions that are obtained can be applied to the design of useful structures or machines in engineering applications.

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